Technical Notes

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Thermal Boundary Layer Due to Sudden Heating of Fluid

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Introduction

UNSTEADY heat-transfer problems related to a closed-cycle, pulsed, high-power laser flow loop are characterized by sudden variations of fluid bulk temperature. Munukutla and Venkataraman[1] discussed the high-power laser flow loop in detail and introduced the associated unsteady heat-transfer problems. The related idealized problem they solved was that of the incompressible flow past a flat plate subjected to a step change in the fluid bulk temperature. The response of the wall heat transfer and thermal boundary-layer thickness to this step change in temperature was calculated using the approximate integral technique. There was a "jump" in the solutions due to the change in the character of the governing differential equation, introduced by the use of the integral technique.

It is, therefore, proposed to solve the flat-plate unsteady heat-transfer problem computationally. The continuity and the momentum equations together with the unsteady energy equation are solved using the Keller-Box method. The accuracy of the solutions is verified by comparison with the steady-state solutions at large times, and the comparison was found to be excellent. Similarity variables proposed in the earlier work in Ref. 1 were found to exist. Empirical formulas for calculating the time-dependent boundary-layer thickness and wall heat transfer are proposed for convenience for use of the computational results by laser flow loop designers. Further discussion on the laser flow loop is given by Knight et al.[2]

Formulation of the Problem

The problem considered here is that of two-dimensional incompressible flow past a flat plate. Initially the fluid and the plate are at the same temperature and there exists a hydrodynamic boundary layer. There is no thermal boundary layer and the wall heat transfer is zero. It is now assumed that there is a step change in the fluid bulk temperature. This situation corresponds to the instantaneous energy deposition by a pulsed E-beam in a laser as described in Ref. 1. The plate temperature is assumed to be constant. It is proposed to calculate the time-dependent thermal boundary-layer growth and wall heat transfer. These are obtained by a computational procedure described in what follows. In view of the incompressible flow assumption, the hydrodynamic boundary layer is undisturbed even when there is a step change in the fluid bulk temperature.

The boundary-layer continuity and momentum equations for the two-dimensional incompressible flow past a flat plate are

$$\frac{\partial u_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{1}$$

$$u_x \frac{\partial u_x}{\partial x} + v_y \frac{\partial u_x}{\partial y} = \nu \frac{\partial^2 u_x}{\partial y^2} \tag{2}$$

and the time-dependent temperature field equation is

$$\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Here u_x and v_y are the x and y components of velocity, and T is the temperature. The symbols ν and α represent the kinematic viscosity and thermal diffusivity of the gas, respectively, and t represents the time. The following nondimensionalization is used for Eqs. 2 and 3:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}\sqrt{Re_L}, \quad U = \frac{u_x}{U_\infty}, \quad V = \frac{v_y}{U_\infty}\sqrt{Re_L}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \tau = \frac{tU_\infty}{L} \quad \text{and} \quad Pr = \frac{\nu}{\alpha} \tag{4}$$

Here L is a representative plate length and U_∞ is the freestream velocity. T_w is the constant wall temperature as well as the initial fluid temperature. At time $t > 0$ there is a step change in fluid temperature from $T = T_w$ to T_∞ while the plate temperature remains constant. Re_L is the Reynolds number based on L.

The stream function ψ is now introduced as

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \tag{5}$$

Further, the following transformation is introduced:

$$\eta = \frac{Y}{\sqrt{X}}, \quad \psi = \sqrt{X} f(X,\eta) \tag{6}$$

The nondimensional momentum equation can be written as

$$f''' + \tfrac{1}{2} ff'' = X\left(f' \frac{\partial f'}{\partial X} - f'' \frac{\partial f}{\partial X}\right) \tag{7}$$

and the nondimensional unsteady temperature field equation can be written as

$$\frac{1}{Pr}\Theta'' + \frac{f}{2}\Theta' = X\left(f' \frac{\partial \Theta}{\partial X} - \Theta' \frac{\partial f}{\partial X} + \frac{\partial \Theta}{\partial \tau}\right) \tag{8}$$

The boundary conditions are

$$f = f' = 0 \quad \text{at} \quad \eta = 0$$
$$f' = 1 \quad \text{as} \quad \eta \to \infty$$

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$$\text{for} \quad \tau = 0, \quad \Theta = 1$$
$$\tau > 0, \quad \eta = 0, \quad \Theta = 1$$
$$\eta \to \infty, \quad \Theta = 0 \tag{9}$$

A two-point finite-difference method, the Keller-Box method[3] is used to solve Eqs. 7–9. Applications of the Keller-Box method to several problems are given by Cebeci and Bradshaw[4] and Cebeci.[5]

According to the Box method, Eqs. (7) and (8) are written as a system of first-order equations. New dependent variables $u(X,\eta)$, $v(X,\eta)$, and $p(X,\tau,\eta)$ are introduced, so that the following system results:

$$u = f', \quad v = u', \quad p = \Theta' \tag{10}$$

$$v' + \frac{fv}{2} = X\left(u\frac{\partial u}{\partial X} - v\frac{\partial f}{\partial X}\right) \tag{11}$$

$$\frac{1}{Pr}p' + \frac{fp}{2} = X\left(u\frac{\partial \Theta}{\partial X} - p\frac{\partial f}{\partial X} + \frac{\partial \Theta}{\partial \tau}\right) \tag{12}$$

It can be seen that Eqs. (11) and (12) correspond to Eqs. (7) and (8), respectively.

In order to use the Keller-Box method, Eqs. (10–12) are cast in a two-point finite-difference form. A net cube (Fig. 1, Ref. 5) is chosen for the difference equations. A smaller net spacing near the wall and larger net spacing away from the wall are used to preserve accuracy and save computaional time. The difference equations needed to approximate Eq. (10) are formulated using the centered difference quotients and are averaged about the midpoint of a face, as suggested by Eqs. (21a, b) in Ref. 5. The finite-difference approximation for Eq. 11 is written for the midpoint of the front face of the cube, since this equation does not involve the time coordinate. The difference equation corresponding to Eq. 12 is centered at the midpoint of the cube, since this equation involves all the three coordinates. The resulting implicit system of nonlinear algebraic equations is then linearized by using Newton's method and solved by the block elimination method.[6]

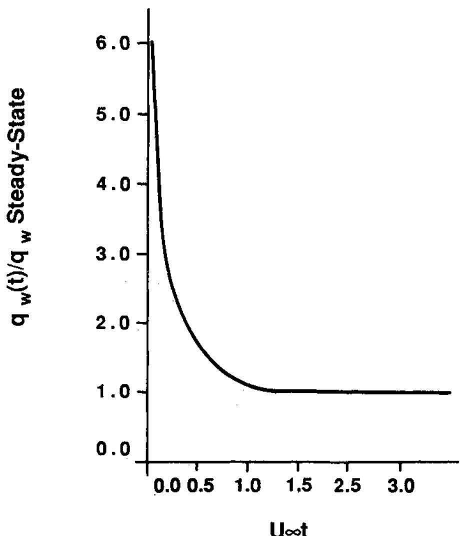

Fig. 1 Growth of thermal boundary layer with time for $Pr = 0.72$.

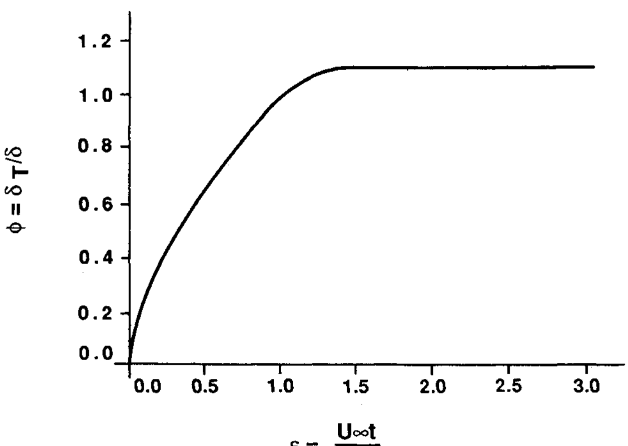

Fig. 2 Wall heat transfer as function of time for $Pr = 0.72$.

Results and Discussion

At any given X, the computations show that the profile of Θ vs η starts developing and does not change for large values of τ indicating that the steady-state solution is reached. When $Pr = 1$ and $\partial/\partial t = 0$ (corresponding to steady state) the equations for U and Θ are identical. In view of the boundary conditions, the solutions for $(1 - U)$ and Θ are, therefore, identical for this case. This information was used to check the accuracy of the computations, which was found to be excellent.

The thermal boundary-layer thickness δ_T and the hydrodynamic boundary-layer thickness δ are defined as

$$U = 0.99 \quad \text{at} \quad \eta = \delta$$
$$\Theta = 0.01 \quad \text{at} \quad \eta = \delta_T$$

In an earlier attempt at solving this unsteady heat-transfer problem using integral technique,[1] it was found that the following similarity parameters existed:

$$\phi = \frac{\delta_T}{\delta} \quad \text{and} \quad \epsilon = \frac{U_\infty t}{x} = \frac{\tau}{X} \tag{13}$$

The present computational results are plotted as ϕ vs ϵ in Fig. 1. This shows that, indeed, a similarity exists. However, this similarity profile is not identical to the one obtained in Ref. 1. This is because of the quasisteady approximation used there and also because of the jump in the solution in Ref. 1.

The ratio of the instantaneous wall heat flux to the steady-state wall heat flux is equal to the ratio of the steady-state thermal boundary-layer thickness to the instantaneous thermal boundary-layer thickness.[1] This is plotted in Fig. 2. It can be seen that, at the instant the fluid bulk temperature increases, the wall heat transfer is infinite since the thermal boundary-layer thickness is zero. The wall heat transfer starts decreasing and reaches the steady-state value as time increases.

Figures 1 and 2 show, in essence, the thermal boundary-layer thickness and wall heat transfer as functions of time at any given x. Empirical curve fits are given for those two parameters as

$$\phi = 0.9273\epsilon^{0.549} \tag{14}$$

$$\frac{q_w(t)}{q_{w\,\text{steady}}} = 1.1474\epsilon^{-0.576} \tag{15}$$

It is hoped that these two equations will help the designer in calculating these unsteady parameters easily.

References

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Structural Modification to Glass-Like Materials Under Laser Irradiation

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I. Introduction

THE scientific questions raised by the coupling of laser radiation into material surfaces have been studied for several years.[1] For glass and glass-like materials there are several mechanisms for laser-induced changes to the material. One of them is bubble formation and growth inside the material while it is softened under the thermal energy deposited by the laser.[2]

For glass-like materials such a fused silica, there is very little dissolved gas, so that bubble growth by absorption of gas cannot be expected to occur. However, there are inhomogeneous quartz materials that are constructed by compressing fibers of quartz together at high temperature and pressure. This process leaves many voids in the material which contain trapped gas. These voids can serve as nucleation sites for bubble growth by expansion of the trapped gas under the laser irradiation. This process of growth will lead to irreversible changes in the glass-like material if the trapped gas is initially "frozen in" out of equilibrium. In this Note we shall address this issue of bubble growth in a glass-like material and the concomitant changes in bulk properties of the material.

II. Bubble Growth for a Single Bubble

We consider a spherical bubble of initial radius R_o containing trapped gas in a glass-like material whose viscosity is $\mu(T)$ where T is the time-varying temperature of the glass around the bubble. The boundary of the bubble $[R(t)]$ grows according to the Rayleigh equation[2,3]

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{1}{\rho_1}\left[p_o\left(\frac{T(t)}{T_o}\right)\left(\frac{R_o}{R}\right)^3-\frac{2\sigma}{R}-4\mu[T(t)]\frac{\dot{R}}{R}\right] \quad (1)$$

In Eq. (1), $\dot{R}=\mathrm{d}R/\mathrm{d}t$, ρ_1 is the (liquid) glass density, p_o is the pressure of the trapped gas at temperature T_o and σ is the surface tension of the bubble. In writing Eq. (1) the external pressure has been neglected. This can be justified either by noting that the laser glass interaction may occur in a vacuum or by noting that for the expected pressures in the bubbles ($p_o \gg 1$ atm) an external pressure of up to 1 atm will be small. Also in Eq. (1) the heat from the glass is taken to flow into the bubble sufficiently fast that the gas temperature in the bubble adiabatically follows the glass temperature. For SiO_2 we take the viscosity coefficient as $\mu(T)=\eta_o e^{Q/T}$ where $Q = 30{,}000$ K and $\eta_o = 10^{-3}$ kg-m/s. The surface tension is taken as $\sigma = 0.3$ Nt/m. The Rayleigh equation must be solved subject to the initial conditions $R(0)=R_o$, $\dot{R}(0)=0$.

The Rayleigh equation was solved numerically by separating Eq. (1) into two first-order equations in time and integrating the coupled set.[4] The temperature of the glass around the bubble will be a function of time as the glass absorbs the laser radiation. In order to be definite, the temperature dependence on time was taken as that given by solving the unsteady one-dimensional energy balance equation for SiO_2 absorbing DF laser radiation.[5] In Fig. 1 a plot of time against bubble radius obtained by solving Eq. (1) is shown for a range of different initial radii. The initial pressure inside the bubble was taken as 5 atm at 300 K. This is a reasonable choice based on the manufacturing process for these materials. Figure 1 indicates that the bubbles did not start growing until after a delay time and then quickly went to their equilibrium radii. The delay in the bubble growth occurs because, below a certain temperature, which we shall call the freezing temperature, the glass is so viscous that no motion of the bubble boundary is possible on the timescale of a few seconds that we are considering. Above this temperature, the glass viscosity decays very rapidly. This freezing temperature is $\simeq 1500$ K. The final bubble radii are based upon the pressure of the trapped gas at $\simeq 1500$ K ($\simeq 25$ atm).

Therefore, from these numerical results, we can deduce that there is a freezing pressure, p_{freeze}, (which is a function of timescale) given approximately by (for a timescale of a few seconds) $p_{\mathrm{freeze}} = p_o(1500/T_o)$. For this freezing pressure the equilibrium solution of Eq. (1) is R_{eq} given by

$$\frac{R_{\mathrm{eq}}}{R_o}=\left(\frac{p_{\mathrm{freeze}}R_o}{2\sigma}\right)^{1/2} \quad (2)$$

Physically, the freezing pressure is the pressure at the freezing temperature above which the glass can flow freely.

III. Equilibrium Bubble Distribution Function

The results of Sec. II show that a set bubble with pressure p_o at T_o in SiO_2 will reach its equilibrium radius given by Eq. (2) during the laser pulse. For a glass-like material of initial volume V_i we define a distribution function for the bubbles by letting $g(R,t)\,\mathrm{d}R$ be the number of bubbles at time t with radii between R and $R+\mathrm{d}R$. The number of bubbles at any time is then

$$N=\int_0^\infty g(R,t)\,\mathrm{d}R \quad (3)$$

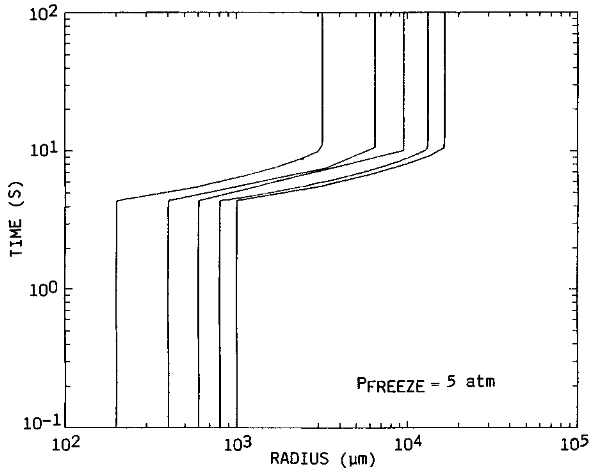

Fig. 1 Time against bubble radius (μm) for the surface temperature profile of a DF 10 s laser pulse in SiO_2.

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